

## The Nature of Science Series #53

### PHYSICS XXI: "THE BIG P AND A"

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Momentum,  $p$ , is the quantity of motion and is equal to the product of the mass of a body times its velocity, or  $p = mv$ .

Acceleration is the rate at which speed changes, or  $a = \frac{v}{t}$ . Note also:  $v = s/t$ .

The Law of Conservation of Momentum states that the total momentum of an isolated system is constant; thus, in mathematical terms:  $(mv)_{\text{before}} = (mv)_{\text{after}}$ , and this reads: "The sum of mass times velocity equals the sum of mass times velocity." This "law" is actually a summary statement of Newton's Third Law of Motion (action-reaction); thus: "In any system (interaction relationship) of bodies not acted upon by an outside (of the system) force, the total momentum ( $mv$ ) remains constant, regardless of any changes which take place in the motions of the individual bodies."

A small body can have a large momentum even if its velocity is large, but a heavy body can have a large momentum even if its velocity is small. The longer a force acts, the greater the effect it produces in changing momentum through changing the velocity of the body. In the previous article we discussed Impulse. When an unbalanced force,  $F_u$ , acts for a length of time,  $t$ , the body acted upon will experience a change in momentum; the product  $F_u t$  is given the special name of impulse; thus: Impulse =  $p$  (or a change in momentum).

Whenever any object is invested with a specific momentum in a given direction, some other body (or bodies) will receive an equal momentum in the opposite direction.

Once more: Newton's Third Law of Motion can be viewed as the "Law of Interaction." Here two equal and opposite forces cancel out by vector addition when they are exerted on the same body. It is this quantity,  $mv$  (mass times velocity) in the formula  $p = mv$ , that is the real measure of the motion of a body:  $mv$  has come to be called "momentum" inasmuch as (1) a fast moving object requires a greater effort (force) to stop it than does the same object moving slowly, and the increase in velocity adds to its total motion, but (2) a massive object moving at a certain velocity requires greater effort (force) to stop it than does a light body moving at the same velocity. The increased mass also adds to the total motion.

Now Impulse ( $I$ ) is the product of force and time; thus:  $I = ft$ . The amount of motion produced by an Impulse has to involve mass and velocity. Try to follow this manipulation: Newton's Second Law of Motion is  $F = ma$ . Since  $F = ma$ , we can substitute  $ma$  for the  $F$  in  $I = Ft$  to get this:  $I = mat$ . Now since  $v = at$ , we can substitute  $v$  for  $at$  in  $I = mat$  to get:  $I = mv$ . Does this last formula look familiar? It should: It is the formula for momentum  $p = mv$ .  $I = mv$  and  $p = mv$ . These two formulae are not equivalent, but are, in fact, the same, so we could write  $p = I$  or  $I = p$ . Or we might write  $p$  or  $I = mv$ .

$I = ft$  means that an Impulse,  $ft$ , applied to a body at rest causes that body to gain a momentum,  $mv$ ,

equal to the Impulse. If, however, the body is already in motion, the application of an impulse brings about a change in momentum equal to the Impulse. In brief, then Impulse = a change in momentum. The units of Impulse must, therefore, be those of force multiplied by those of time ( $I = ft$ ), or those of mass multiplied by those of velocity ( $I = mv$ ).

We can now state the Law of Conservation of Momentum this way:  $mv = ft$ , i.e., the momentum acquired by a body (mass x velocity) is equal to the force exerted by the spring multiplied by the time during which the force acts (force x time). Spring here refers to a compressed spring by a force.

In effect, we have just defined force in terms of momentum; thus, "Force is the rate of change of momentum of a body on which the force acts."

One last consideration of Newton's Three Laws of Motion:

Law #1: In order to change a body's momentum or direction of motion, an outside force must be applied.

Law #2: Such a change is directly proportional to the force and the time over which it is applied ( $F = mv$ ).

Law #3: A recoil obtains when two bodies acquire equal and opposite momenta.

In sum, we can thus say that the First and Third Laws are included in the Law of Conservation of Momentum, and the Second Law is equivalent to the definition of force.

What is the relationship between acceleration and momentum? Simply this: When a body accelerates, its velocity changes, therefore, acceleration always produces a change in momentum. "Accelerates" is from the latin meaning "to add speed." Acceleration, then, is the rate of change of velocity.

What did we say about acceleration above? We said that it is the rate at which speed changes. We can also define it as the time - rate of change of velocity. Then the formula for average acceleration becomes this:  $a = \frac{v_f - v_o}{t}$

Except in special cases, the velocity of a moving body changes continuously as the motion proceeds (i.e., with time). Thus  $v = at$  from one point to the next point. When this results, the body is said to move with accelerated motion, or to have an acceleration.

In article #49 of this series, in the discussion of mass, I wrote the formula:  $m = F/a$ . In every case, the direction of the acceleration of a body is the same as that of the force. This is true whether the body is initially at rest or is moving in any direction and with any speed. Thus, for any given body (here read "mass"), the ratio of the magnitude of the force and that of the acceleration is always the same, or is a constant:  $F/a$  (the "constant" for any given body). This constant ratio of  $F$  to  $a$  is called "mass."

To produce an acceleration, there must be an unbalanced force operating on a body. Anything which requires a force to cause it to accelerate has mass; the size of mass it has would be proportional to the quantity  $F/a$ , which is derived from  $F/m$  or  $m F/a$  ( ). This means that for a given body-mass -- the larger the force, the larger the acceleration. Likewise, for a given value of force -- the larger the mass, the smaller the acceleration.

Acceleration depends on the amount of inertia possessed by a body and inertia depends on the amount

of mass the body has. The larger the initial resistance to motion (i.e., the greater is the amount of mass, hence, the greater is the inertia of the mass), the smaller will be the acceleration. Since mass is the quantitative measure of inertia, we see that the acceleration is inversely proportional to the mass of the body, or symbolically:  $a \propto \frac{1}{m}$ . The amount of motion produced by an applied force on a given mass depends on the magnitude of the force; hence, the larger the force, the larger will be the acceleration, or symbolically:  $a \propto F$ .

If a  $F$  and a  $a$ , then we can combine these equations thusly:  $F \propto ma$ . By selecting the correct choice of proportionality factors and units, we can write:  $F = ma$  (The Second Law of you know who).

Since acceleration takes place whenever velocity is changed and since velocity is a vector quantity, an unbalanced force can either cause (1) a change in the magnitude of the velocity of a body, or (2) a change in the direction of a body. Combinations of changes in speed and direction are also possible. Hence, a body moving with a constant velocity (not constant acceleration) through space will continue to do so unless an unbalanced force is applied to it because such a body is not being positively or negatively accelerated. (I snuck this one in!).

And lastly: Acceleration means a change in velocity, and even a change in direction constitutes a change in velocity. Thus, a body which has the direction of its motion changed is actually accelerated even though its speed does not change. It, therefore, follows that an unbalanced force is required to produce this acceleration.

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